

Original Research

Exact traveling wave solutions of the Zoomeron equation via the modified (G'/G) -expansion method

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Received: 02/09/2023; Accepted: 12/09/2024; Online available: 30/09/2024.

Abstract

The traveling wave solutions to the nonlinear Zoomeron equation are obtained in this study by the use of well-known mathematical systems, such as the modified (G'/G) -expansion technique. Twenty mathematical solutions that are clearly hyperbolic, trigonometric, and irrational are presented. We depict the two dimensional, three dimensional, and contour shapes of our discovered solutions using Maple software. The mathematical systems that produce the king type shape, soliton solutions, bell shape wave profile, and periodic traveling wave profile are depicted here in a thoughtful and clear manner. The originality of our approach is demonstrated by comparing our return to that obtained in another research that has been published. In the fields of applied mathematics, physics, and engineering, these systems are also able to express a diversity of results for other fractional models.

Keywords: Zoomeron equation, modified (G'/G) -expansion scheme, mathematical solutions, NPDEs, king wave, bell wave

Introduction

Albert Einstein once said, "The most incomprehensible thing about the world is that it is at all comprehensible. But how do we fully understand incomprehensible things?" In this sense, nonlinear science offers some hints [1]. The environment we live in is intrinsically nonlinear. In several scientific disciplines, such as fluid mechanics, solid-state physics, plasma physics, plasma waves, and biology, nonlinear evolution equations (NEEs) are frequently employed as models to describe complicated physical events. Finding the traveling wave solutions for those models is one of the fundamental physics issues. In particular, a variety of techniques have been used to investigate distinct physical model solutions that are modeled by nonlinear partial differential equations (NPDEs). Stability and convergence in the numerical approaches should be taken into consideration, in order to prevent inconsistent or unsuitable results. Though excellent investigative and semi-analytical techniques have been developed recently to be employed for explaining NPDEs, notably the Exp(-Phi)-Expansion process [2-5], Bifurcation Analysis [6], the unified technique [7], Sine-Gordon expansion method [8], Kudryashov schemes [9], Jacobi elliptic task technique [10], the Jacobi elliptic ansatz technique [11], fractional iteration algorithm [12,13], variation of expansion method [14], modified decomposition schemes [15], the hyperbolic and exponential ansatz method [16], natural transformation technique [17], Hirota's simple schemes [18,19], the modified extended tanh expansion system [20], and significantly more. In order to convey the reasonability and simplicity of the cycle, we instrument the modified -expansion schemes in the current study to produce accurate solutions to the zoomeron equation. Thus, by fitting transformation, we can simply convert partial request nonlinear population models into NPDE or NODE, which explains why anyone knowledgeable with partial analytics will have no trouble doing so. The main benefit of this cycle over other designs is that it contributes more innovative precise solutions, including additional independent factors, and we also produce some little novel outcomes. The exact reactions are crucial in disclosing the key element of the real events. In addition to its considerable significance, fractional order nonlinear population's particular responses. The first part of this stream item introduces the possibility of the research. The second portion is an investigation of the improved expansion scheme on numerous levels. In the third section, we'll use the method under consideration to obtain the Zoomeron equation's solutions. We will present multiple computer simulations of the resulting solution in the fourth segment. In the final section, the conclusion is prearranged.

The Modified (G'/G)-expansion technique

We are considering:

$$Y(u, u_x, u_{xx}, u_t, u_{tt}, u_{xt}, \dots) = 0,$$

Here, Y is a polynomial in u .

Section I: Appliance the traveling adjustable:

$$u = u(x, t) = u(\xi), \xi = \sigma_3(x - \theta t), \quad (2.2)$$

Where σ_3 and θ are a constant to be determined later. Executing (2.2) into (2.1), we get:

$$S(u, \sigma_3 u', \sigma_3^2 u'', -\sigma_3 \theta u', \dots) = 0 \quad (2.3)$$

Section II: Considering the ansatz form:

$$u(\xi) = \sum_{i=-N}^N \theta_i \Delta^i$$

where $\Delta = \left(\frac{G'}{G} + \frac{\lambda}{2} \right)$, $|A_{-N}| + |A_N| \neq 0$ and $G = G(\xi)$ fulfils the relation

$$G'' + \lambda G' + \mu G = 0 \quad (2.5)$$

where $\theta_i (\pm 1, \pm 2, \dots, \pm N)$, λ and μ are coefficient constants. By applying the principle of homogeneous balance to equation (2.3), it is possible to ascertain the positive integer. We can determine from equation (2.5) that

$$\Delta = r - \Delta^2,$$

Where $r = \frac{\lambda^2 - 4\mu}{4}$ and r is intended through λ and μ . Thus, Δ mollifies (2.6), which yields

$$\Delta = \begin{cases} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi\right), & \lambda^2 - 4\mu > 0; \\ \frac{\sqrt{\lambda^2 - 4\mu}}{2} \coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi\right), & \lambda^2 - 4\mu > 0; \\ \frac{1}{\xi}, & \lambda^2 - 4\mu = 0; \\ -\frac{\sqrt{\lambda^2 - 4\mu}}{2} \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2} \xi\right), & \lambda^2 - 4\mu < 0; \\ -\frac{\sqrt{\lambda^2 - 4\mu}}{2} \cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2} \xi\right), & \lambda^2 - 4\mu < 0. \end{cases}$$

Section III: The left-hand side of (2.3) is transformed into a polynomial in by putting (2.5), (2.4), and (2.3) into practice and gathering all terms with the same order of together. By setting all of the polynomial's coefficients to zero, we may create a system of algebraic equations that we can solve to determine the values of the method under study.

Application of the Modified (G'/G)-expansion scheme

The improved (G'/G)-expansion approach is used in this subsection to resolve the Zoomeron equation in the usage.

$$\left(\frac{u_{xy}}{u}\right)_{tt} - \left(\frac{u_{xy}}{u}\right)_{xx} + 2(u^2)_{xt} = 0 \quad (3.1)$$

where the relative wave mood's amplitude is denoted by $u(x, y, t)$. This formula belongs to the class of incognito evolution. Calogero and Degasperis introduced the equation [21]. By means of the wave variable

$$u(x, y, t) = U(\xi), \xi = x + sy - wt,$$

Equation (3.1) is carried to an ODE

$$s(w^2 - 1)U'' - 2wU^3 - RU = 0, \quad (3.2)$$

Where R is the integration constant and the prime indicates the derivation with respect to ξ . $N = 1$ is obtained by balancing the highest order derivative term u'' of (3.2) with the nonlinear term u^3 . According to modified (G'/G) expansion way,

Now the solution of (3.2) is,

$$U(\xi) = \theta_{-1}\Delta^{-1} + \theta_0 + \theta_1\Delta \quad (3.3)$$

Putting (3.3) in (3.2) with the help of the proposed methods we get, the following set of solutions:

Case-1:

$$S = \pm \sqrt{\frac{R}{W(\lambda^2 - 4\mu)}} \frac{W}{W^2 - 1}, \theta_0 = 0, \theta_{-1} = \pm \sqrt{\frac{R}{W(\lambda^2 - 4\mu)}}, \theta_1 = \pm \frac{1}{4} \frac{\sqrt{R(\lambda^2 - 4\mu)}}{W}$$

Substituting the values of case (1) into (3.3) then we achieve,

$$U_1(\xi) = \pm \sqrt{\frac{R}{W}} \frac{1}{(\lambda^2 - 4\mu)} \left[2 \left\{ \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right\}^{-1} + (\lambda^2 - 4\mu)^2 \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right] \quad (3.4)$$

$$U_2(\xi) = \pm \sqrt{\frac{R}{W}} \frac{1}{(\lambda^2 - 4\mu)} \left[2 \left\{ \coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right\}^{-1} + (\lambda^2 - 4\mu)^2 \coth\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi\right) \right] \quad (3.5)$$

$$U_3(\xi) = \pm \sqrt{\frac{R}{W}} \left[\frac{1}{\sqrt{\lambda^2 - 4\mu}} \left\{ \frac{4\xi^2 + \lambda^2 - 4\mu}{4\xi} \right\} \right] \quad (3.6)$$

$$U_4(\xi) = \pm \sqrt{\frac{R}{W}} \frac{1}{\sqrt{\lambda^2 - 4\mu}} \left[2 \left\{ \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) \right\}^{-1} + (\lambda^2 - 4\mu)^2 \tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) \right] \quad (3.7)$$

$$U_5(\xi) = \pm \sqrt{\frac{R}{W}} \frac{1}{\sqrt{\lambda^2 - 4\mu}} \left[2 \left\{ \cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) \right\}^{-1} + (\lambda^2 - 4\mu)^2 \cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) \right] \quad (3.8)$$

Case-2:

$$S = -\frac{2R}{W^2 \lambda^2 - 4W^2 \mu - \lambda^2 + 4\mu}, \theta_0 = 0, \theta_{-1} = \pm \sqrt{\frac{2R}{W(\lambda^2 - 4\mu)}} i, \theta_1 = 0$$

Substituting the values of case (2) into (3.3) then we achieve,

$$U_6(\xi) = \pm \left[i \sqrt{\frac{R}{2W}} \left\{ \tanh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \right) \xi \right\}^{-1} \right] \quad (3.9)$$

$$U_7(\xi) = \pm \left[i \sqrt{\frac{R}{2W}} \left\{ \coth \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \right) \xi \right\}^{-1} \right] \quad (3.10)$$

$$U_8(\xi) = \pm \left[i \sqrt{\frac{2R}{W(\lambda^2 - 4\mu)}} \xi \right] \quad (3.11)$$

$$U_9(\xi) = \pm \left[i \sqrt{\frac{R}{2W}} \left\{ \tan \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \right) \xi \right\}^{-1} \right] \quad (3.12)$$

$$U_{10}(\xi) = \pm \left[i \sqrt{\frac{R}{2W}} \left\{ \cot \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \right) \xi \right\}^{-1} \right] \quad (3.13)$$

Case-3

$$S = -\frac{2R}{W^2 \lambda^2 - 4W^2 \mu - \lambda^2 + 4\mu}, \theta_0 = 0, \theta_{-1} = 0, \theta_1 = \pm \sqrt{\frac{R(4\mu - \lambda^2)}{8W}}$$

Substituting the values of case (3) into (3.3) then we achieve,

$$U_{11}(\xi) = \pm \left[\sqrt{\frac{R(4\mu - \lambda^2)}{8W}} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left\{ \tanh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \right) \xi^{-1} \right\} \right] \quad (3.14)$$

$$U_{12}(\xi) = \pm \left[\sqrt{\frac{R(4\mu - \lambda^2)}{8W}} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left\{ \coth \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \right) \xi^{-1} \right\} \right] \quad (3.15)$$

$$U_{13}(\xi) = \pm \left[\sqrt{\frac{R(4\mu - \lambda^2)}{8W}} \xi^{-1} \right] \quad (3.16)$$

$$U_{14}(\xi) = \pm \left[\sqrt{\frac{R(4\mu - \lambda^2)}{8W}} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left\{ \tan \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \right) \xi^{-1} \right\} \right] \quad (3.17)$$

$$U_{15}(\xi) = \pm \left[\sqrt{\frac{R(4\mu - \lambda^2)}{8W}} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left\{ \cot \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \right) \xi^{-1} \right\} \right] \quad (3.18)$$

Case-4:

$$S = \pm \sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}} \frac{W}{W^2 - 1} i, \theta_0 = 0, \theta_{-1} = \sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}} i, \theta_1 = -\frac{1}{8} \frac{R}{W} \sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}} i$$

Substituting the values of case (4) into (3.3) then we achieve,

$$U_{16}(\xi) = \pm \frac{i}{2} \left\{ \tanh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right\}^{-1} \left[\sqrt{\frac{R}{2W}} + \frac{1}{8} \frac{R}{W} \frac{1}{\sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}}} \left(\sqrt{\lambda^2 - 4\mu} \right) \right] \quad (3.19)$$

$$U_{17}(\xi) = \pm \frac{i}{2} \left\{ \coth \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right) \right\}^{-1} \left[\sqrt{\frac{R}{2W}} + \frac{1}{8} \frac{R}{W} \frac{1}{\sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}}} \left(\sqrt{\lambda^2 - 4\mu} \right) \right] \quad (3.20)$$

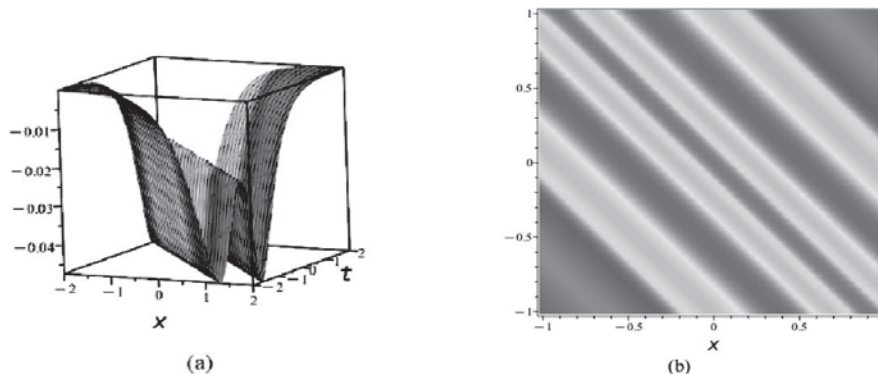
$$U_{18}(\xi) = \pm i \left(1 + \frac{R}{8W} \right) \left[\sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}} \frac{1}{\xi} \right] \quad (3.21)$$

$$U_{19}(\xi) = \pm \frac{i}{2} \left\{ \tan \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right\}^{-1} \left[\sqrt{\frac{R}{2W}} + \frac{1}{8} \frac{R}{W} \frac{1}{\sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}}} \left(\sqrt{\lambda^2 - 4\mu} \right) \right] \quad (3.22)$$

$$U_{20}(\xi) = \pm \frac{i}{2} \left\{ \cot \left(\frac{\sqrt{4\mu - \lambda^2}}{2} \xi \right) \right\}^{-1} \left[\sqrt{\frac{R}{2W}} + \frac{1}{8} \frac{R}{W} \frac{1}{\sqrt{\frac{R}{2W(\lambda^2 - 4\mu)}}} \left(\sqrt{\lambda^2 - 4\mu} \right) \right] \quad (3.23)$$

Graphical Representation

Graphs are a useful tool for advising and for calling problems' solutions calmly. A blueprint is a visible depiction of incomplete or imperfect solutions, or other data, typically used for allusive purposes. When assuming addition in routine activity, we need the fundamental capacity of using graphs effectively. We will discuss the diagrammatic depiction of the discovered solutions in this segment:



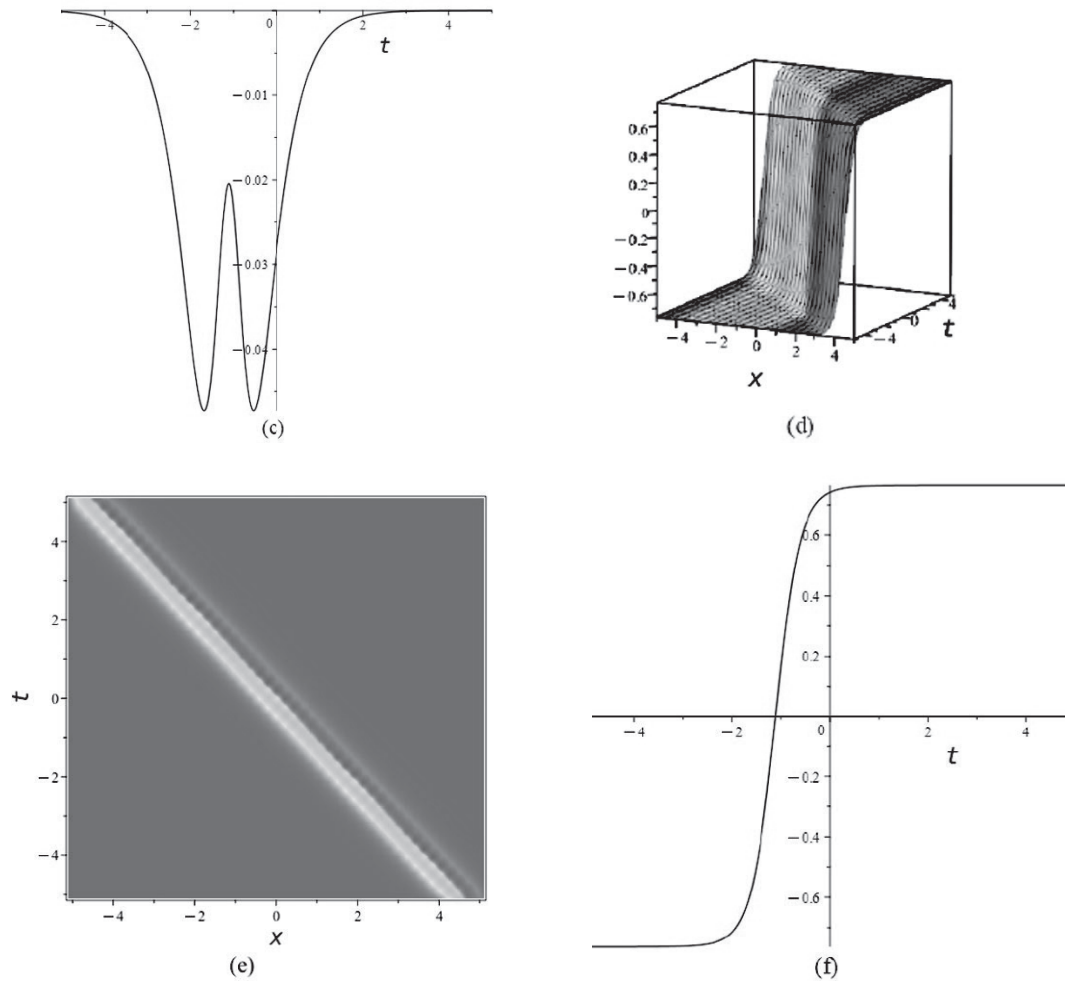
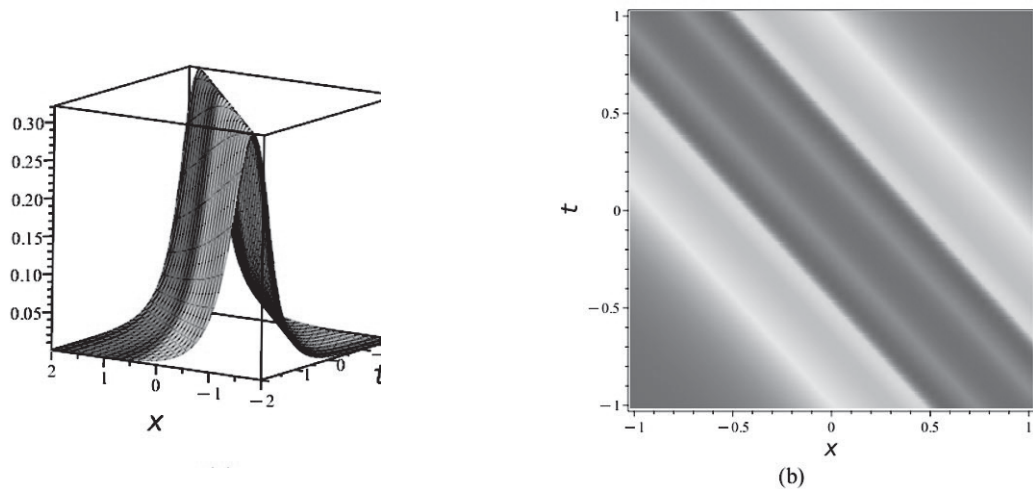


Fig1: The graphical representation of Eq.(3.5): (a) real 3D shape, (b) real density plot, (c) real 2D shape, (d) complex 3D shape, (e) complex density plot and (f) complex 2D shape.



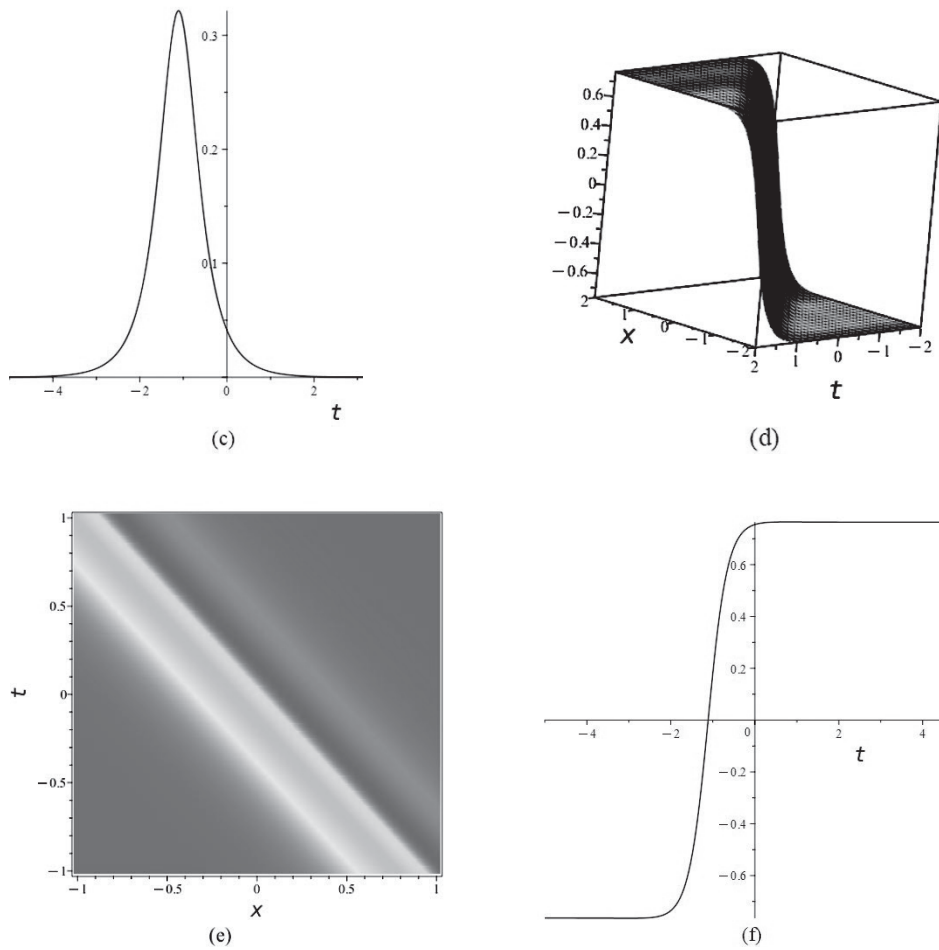
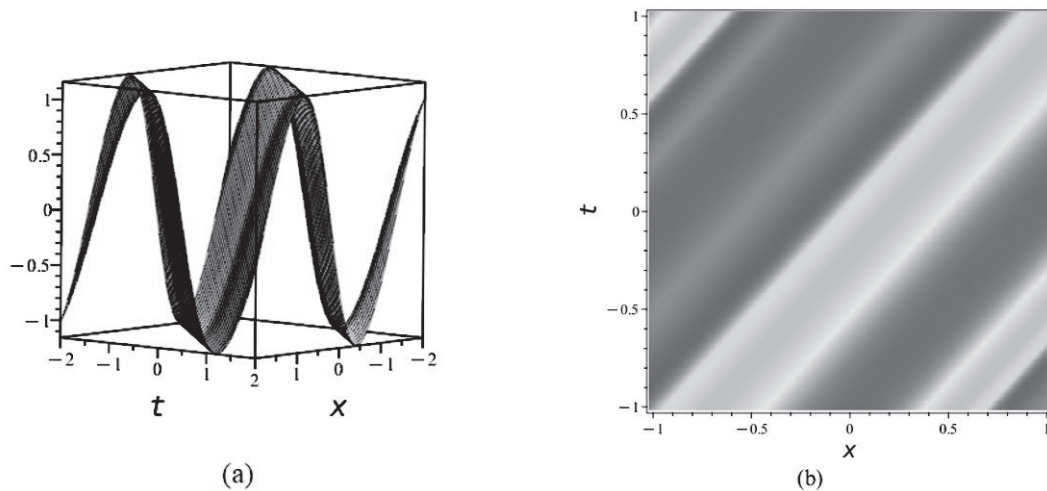


Fig 2: The graphical illustration of Eq. (3.4): (a) real 3D profile, (b) real density plot, (c) real 2D profile, (d) imaginary 3D shape, (e) imaginary density plot and (f) imaginary 2D shape.



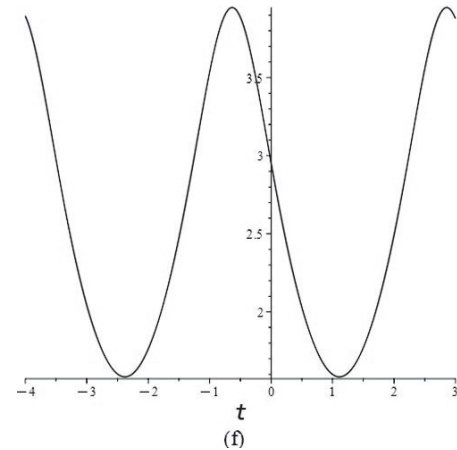
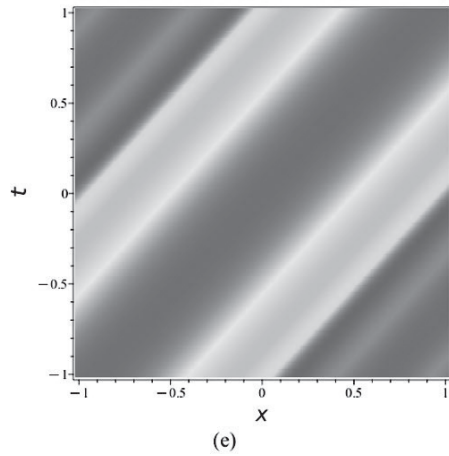
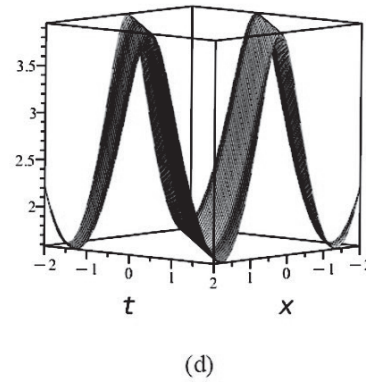
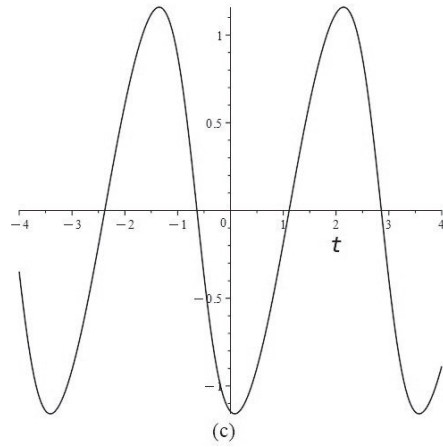
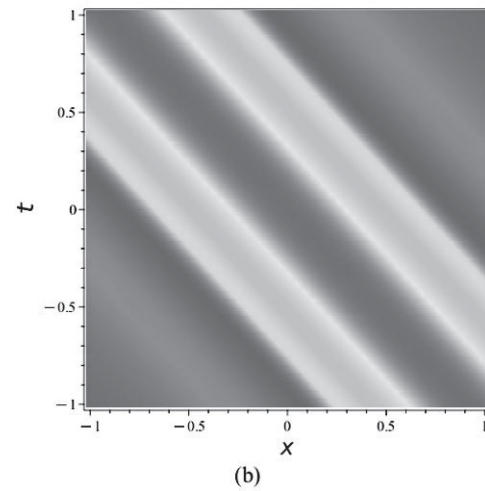
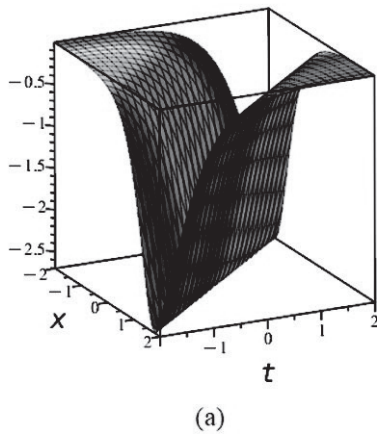


Fig 3: The graphical depiction of Eq. (3.7): (a) real 3D form, (b) real density plot, (c) real 2D shape, (d) complex 3D form, (e) complex density plot and (f) complex 2D shape.



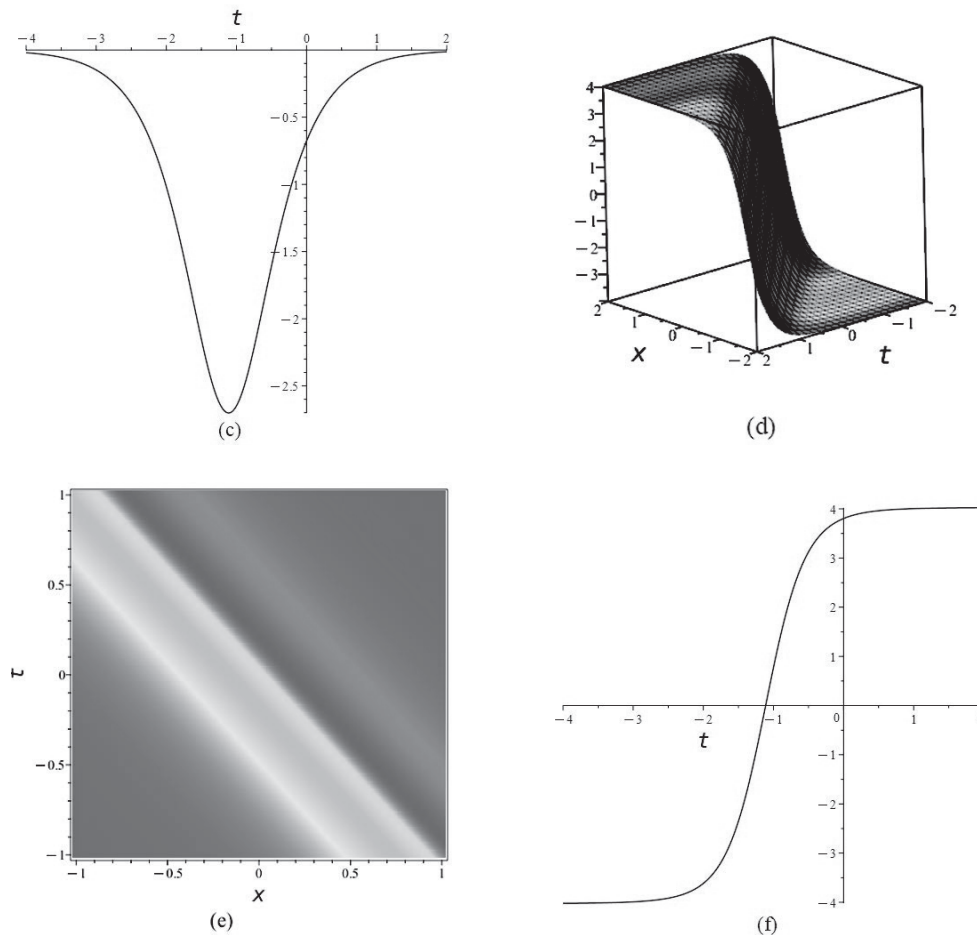


Fig 4: The graphical picture of Eq. (3.5): (a) real 3D nature, (b) real density plot, (c) real 2D nature, (d) complex 3D nature, (e) complex density plot and (f) complex 2D nature .

Numerical Simulations via the Modified (G'/G)-expansion technique

In the ongoing segment, we have offered various numerical simulation using the proposed approaches. To make sense of the powerful exhibitions of the responses procured in segment 3. Fig. 1-4 show the visual portrayals of a few chose computing consequences of the issue got using the concentrated-on strategy. They are presented underneath.

Fig.1 exhibits the unique presentation of Eq. (3.5) using the parameters $\lambda = -4$, $\mu = 2$, $R = 2$, $W = 0.5$, $y = 0.5$. Specifically, Figure 1 shows the 3D form (real and complex), 2D form (real and complex) and density form (real and complex) of Eq. (3.5). The real part of this shape addresses the W - wave profile, and complex part represents the anti-king wave profile. The result characteristics of Eq. (3.4) are presented in Fig.2 using $\lambda = 3$, $\mu = 1$, $R = 0.5$, $W = -0.9$, $y = 0.5$. This shape addresses the bell shape and king wave profile. The nature of the result of Eq. (3.7) are shown in Fig.3 using $\lambda = -2$, $\mu = 2$, $R = 0.5$, $W = 0.9$, $y = 0.5$. This shape addresses the periodic wave profile. The clarification features of Eq. (3.5) are exhibited in Fig.4 using $\lambda = 3$, $\mu = 1$, $R = 0.5$, $W = -0.9$, $y = 0.5$. This shape addresses cusp wave of multiple wings shape and king wave profile.

Conclusion

By using the suggested methodology, our assessment has inspected the novel computerized responses of the Zoomeron equation. In hyperbola, rational and trigonometric equations represented in king form profile, singular

king shape, periodic wave and bell shape wave profile, several novel computational outcomes have been attained. The distinctiveness of our investigation is demonstrated by comparing our observed reactions to those found in recently written research articles. The pre-owned technique demonstration reveals that these strategy suitability, impact, and ability to work with different nonlinear models call for further investigation.

Authors Contribution's

Kanak Chandra Roy: Writing - original draft, Methodology, Formal analysis. Md. Sabur Uddin: Resources, Methodology, Formal analysis, Validation, Writing - original draft, review. Dilruba Akter Pansi: Methodology, Software, Malati Mazumder: Writing - original draft. Asha Moni Akter: Wrote paper & editing.

Acknowledgements

The authors would like to express their sincere gratitude to the all-faculty member of Applied Mathematics, Gono Bishwabidyalay for their constant support to complete this article.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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