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Solution of Maxwell's Equations Implementing Free-Space Set

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Abstract

The equation of Maxwell's, which designate the electromagnetic force, are often a series of four partial differential equations. The historical context of Maxwell's equations was originally covered in this essay. Then, using known works by Gauss' Law, Faraday's Law, and Ampere's Law, we were able to derive Maxwell's equations. In addition, we have implemented the general set and free-space set to interpret the Maxwell's equation's solution.

Keywords: *Maxwell's equations, magnetism, magnetic field, magnetic flux, electric field*

1. Introduction:

James Clerk Maxwell (1831-1879) is regarded as the greatest theoretical physicist of the 19th century. Although his lifetime was very short, Maxwell not only formulated a complete electromagnetic theory, represented by Maxwell's equations, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn's rings. Maxwell brought together all the work that had been done by brilliant physicists, Oersted, Coulomb, Gauss and Faraday. They had shown that electric currents can generate magnetic fields and that moving magnets can generate electric currents. Maxwell's equations completely define the evolution of the electromagnetic field. Also given a complete specification of an electromagnetic systems boundary conditions and constitutive relationships. That is data defining the materials within the system by specifying the relationships between the electric/ magnetic field and electric displacement/ magnetic induction for which we can calculated the electromagnetic field at all points in the system at any time. The combination of Maxwell's equations, boundary conditions and constitutive relations show everything that can be experimentally measured about electromagnetic effects [1]. Maxwell's equations have typically been presented in accordance with the historical technique of first studying electrostatics and magnetostatics. Faraday's induction law is then introduced to account for quasistatic phenomena. We finally incorporate the displacement current to assure charge conservation and to establish

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the Ampere-Maxwell law, which completes the set of equations known as Maxwell's equations [2]. The reader may question why an axiomatic approach to Maxwell's equations is not commonly available in undergraduate textbooks, despite the fact that axiomatic presentations of quantum physics and general relativity can be found [3]. One advantage of the axiomatic approach is that it provides the quickest route to the essence of a theory and allows for more rigorous formulation [4]. Recognizing the main postulates underlying Maxwell's equations is the fundamental difficulty for an axiomatic presentation of Maxwell's equations. One of these postulates cannot be avoided: charge conservation, as stated by the continuity equation for charge and current densities. The electromagnetic force is described by a set of four partial differential equations called Maxwell's equations. James Clerk Maxwell, a mathematician, developed them and initially published them in 1861 and 1862 [5].

It should be emphasized, however, that magnetism and electricity were formerly thought to be independent sensations and were just recently proven to be different manifestations of a single major source. In the eighteenth century, physicists such as Charles-Augustin Coulomb (1736-1806) experimented with electric charges and observed how they attract or repel one other via electric fields - in current vernacular, electromagnetic fields, they worked out applications of Gauss' law: $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$, which is the field of electrostatics. One hundred years later, physicists researching magnetism manipulated currents and investigated how they interact with one another via magnetic fields. Oersted observed that wires carrying electric currents deflected a compass needle put near them around 1820. Biot and Savart, and later Ampere, created strict principles that connected the direction and strength of a magnetic field to the streams that produced it. In modern language they sketched out consumptions of Ampere's law: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$. This is the field of magnetostatics [6]. Individually, the four equations are named Gauss’s law, Gauss’s law for magnetism, Faraday’s law and Ampere’s law. The equations look like this:

i. $\nabla.\vec{D} = \rho_v$

ii. $\nabla.\vec{B} = 0$

iii. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

iv. $\nabla \times \vec{H} = \vec{J}^{\,i} + \frac{\partial \vec{D}}{\partial t}$

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where $\vec{D}$ = displacement vector, ρ_v = charge density, $\vec{B}$ = magnetic field induction , $\vec{E}$ = electric intensity, $\vec{J}^i$ = current density, $\vec{H}$ = magnetic field intensity

while utilizing these equations requires integrating (calculus), we can still discuss theoretically what each law represents and how it is used:

Gauss's Law: An electric field is created by electric charges. The electric flux through a closed circuit is proportional to the charge enclosed. Gauss's Law for Magnetism: Monopolies do not exist. The magnetic flux and the quantitative applications of Faraday's law across a closed surface are both zero. Faraday's Law: Magnetic fields that change over time generate an electric field. Ampere's Law: A magnetic field is produced by steady currents and time-varying electric fields (the latter due to Maxwell's correction). Diener et al. [7] used experimental evidence of charge conservation, the fact that electromagnetic waves propagate at the speed of light c, and the need of Galilean invariance of the Lorentz force for low velocities to develop the Maxwell equations and introduce electrodynamic units. Munz and Sonnendrücker [8] has described the Maxwell's equations in free space form a linear hyperbolic system. The equations of Magnetohydrodynamics (MHD) describe the evolution of a plasma, which is a gas of charged particles considered as a single fluid. The ideal MHD equations result from the conservation of mass, momentum and energy, of the fluid along with the appropriately reduced Maxwell's equations. Francisco and Duarte [9] presented the entire spectrum of traditional electromagnetic phenomena. Light can be defined classically as waves of electromagnetic radiation. As a result, Maxwell equations are highly useful for illustrating a variety of light qualities, including polarization. Charles R. Mac Cluer (2021)[10] found that the differential homogeneity of Maxwell's equations exist in free space and each solution can be perturbed in infinitely many ways into a physically unreasonable solution. He also found that, the traditional first example the uniform plane wave must be considered as only a metaphor.

2. Derivation of Maxwell's Equations.

2.1 First Equation:

From Gauss's law in dielectrics

$$\int_S \vec{D}.d\vec{a} = q_i$$

where q_i is the whole charge covered by the surface *S*. It may be written as a volume integral

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$$q_i = \int_v \rho_v d\tau$$

where *v* represents the volume encompassed by *S*. Then

$$\int_S \vec{D}.d\vec{a} = \int_v \rho_v d\tau$$

But according to Gauss's divergence theorem in vectors

$$\int_S \vec{D}.d\vec{a} = \int_v \left(Div\vec{D}\right) d\tau$$

Thus, we have

$$\int_V \nabla.\vec{D} d\tau = \int_v \rho_v d\tau$$

The volume under consideration is completely arbitrary, so

$$\nabla.\vec{D} = \rho_v$$

which is the first equations of Maxwell's. It asserts that at any position, the free volume charge density is equal to the divergence of electric displacement at that point.

2.2 Second Equation:

An electric charge sets up an electric field that can after other electric charges. In analogy with the electric field, the theory does not prevent us from writing for the magnetic field.

$$\nabla.\vec{B} = \rho_m$$

Where ρ_m is "magnetic charge density" due to "magnetic charges". Such isolated magnetic charges are called magnetic monopoles by Manfredi [11].

Although theory permits the existence of magnetic monopoles, however isolated magnetic monopole have not ever been found despite intense experimental searches. So,

$$\int_S \vec{B}.d\vec{a} = 0$$

Magnetic lines of force are thus either closed curves or elliptical curves. In other words, the quantity of magnetic positions of force leaving and entering a few given closed surface is correctly equal. Using Gauss's divergence theorem, one finds

$$\int_V \nabla.\vec{B} d\tau = 0$$

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Since the volume is arbitrary, it implies

$$\nabla.\vec{B}=0$$

The presence of isolated magnetic charges (monopoles) was hypothesized in 1931 by theoretical physicists P. Dirac based on quantum mechanics and symmetry arguments. Following Dirac's prediction, enormous practical accelerators were used to seek for magnetic monopoles, but none of them found any evidence of their existence. Furthermore, new attempts to unify the principles of physics by combining the strong, weak, and electromagnetic forces into a single framework have reignited interest in magnetic monopoles.

2.3 Third Equation:

Faraday's law of electromagnetic stimulation states that, the e.m.f (e) induced in a closed loop is equal to negative rate of change of magnetic flus i.e.

But magnetic flux $\phi=\int_S \vec{B}.d\vec{a}$

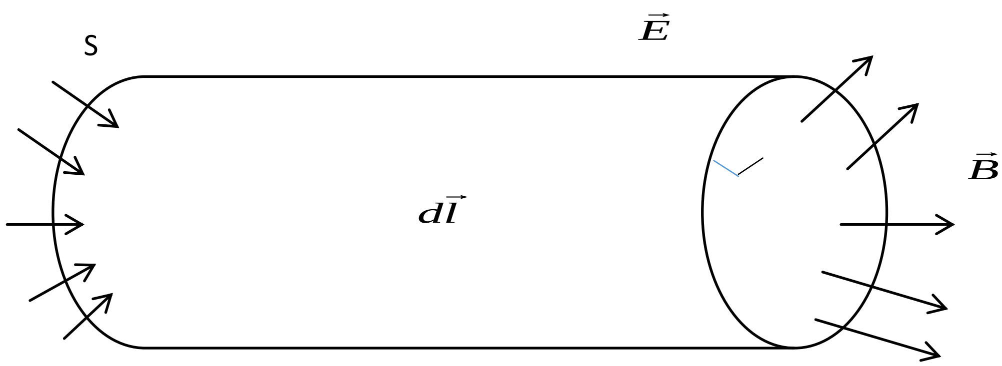

Figure1- Pertaining to Maxwell's curl equation for $\vec{E}$

S is several undefended surface with a loop as its limit.

Furthermore, e.m.f. is the effort done in transporting a unit (positive) charge through the closed loop C.

or, $e=\oint_C \vec{E}.d\vec{l}$

Therefore

$$\oint \vec{E}.d\vec{l}=\frac{\partial}{\partial t}\oint_S \vec{B}.d\vec{a}$$

Using Stoke's theorem of vectors

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$$\oint_C \vec{E}.d\vec{l} = \int_S \left(\nabla \times \vec{E}\right).d\vec{a}$$

Consequently

$$\int_S \left(\nabla \times \vec{E}\right).d\vec{a} = -\frac{\partial}{\partial t}\int_S \vec{B}.d\vec{a}$$

As the surface S is completely arbitrary,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This expresses the law of Faraday's in differential method.

2.4 Fourth Equation:

The line integral of vector $\vec{H}(\vec{r},t)$ in a closed channel I equals the conduction current I encompassed by the path, according to Ampere's equation. i.e.

$$\int_C \vec{H}.d\vec{l} = I \quad (1)$$

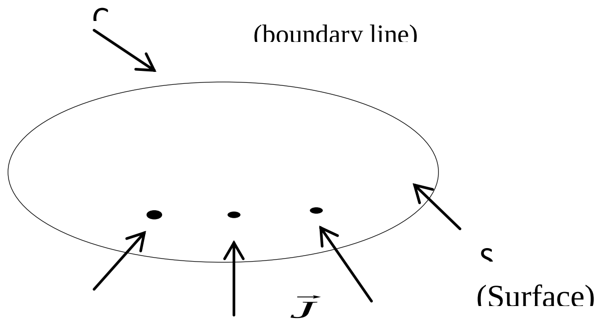

Figure 2- directions for Ampere's Circuital Law

Where,

$$I = \int_S \vec{J}^{\,i}.d\vec{a} \quad (2)$$

This holds for steady state conditions i.e. for

$$\int_V \nabla.\vec{J}^{\,i} d\tau = 0$$

Or, $\nabla.\vec{J}^{\,i} = 0$ (3)

(i.e. when there are no sources of sinks in the volume element.)

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A vector satisfying condition is called a solenoidal vector.

Now, from the concept of displacement current, we know that from time varying fields we have to add displacement current I_d on the R.H.S. of equation (2), which is given by

$$I_d = \frac{\partial}{\partial t}\int_S \vec{D}.d\vec{a} \qquad (4)$$

Therefore,

$$\int_C \vec{H}.d\vec{l} = \int_S \left(J^i + \frac{\partial \vec{D}}{\partial t} \right).d\vec{a} \qquad (5)$$

Using Stoke's theorem on the L.H.S. we get

$$\int_C \vec{H}.d\vec{l} = \int_S \left(\nabla \times \vec{H}\right).d\vec{a} = \int_S \left(J^i + \frac{\partial \vec{D}}{\partial t} \right).d\vec{a}$$

Therefore,

$$\nabla \times \vec{H} = \vec{J}^i + \frac{\partial \vec{D}}{\partial t} \qquad (6)$$

The magnetomotive force (m.m.f) in a closed path is equal to the conduction current plus the time derivative of electric displacement over the surface S limited by the path C, according to Equation (5). Maxwell's version of Ampere's law is thus Equation (5).

The equation of continuity

$$\nabla.\vec{J}^i = -\frac{\partial \rho}{\partial t} \text{ or } \nabla.\vec{J}^i + \frac{\partial \rho}{\partial t} = 0$$

Is also included in Maxwell's equations. Taking the divergence of equation (6) results in

$$\nabla.\left(\nabla \times \vec{H}\right) = \nabla.\vec{J}^i + \nabla.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

The L.H.S. of this equation is zero since the divergence of a vector's curl is always zero.

So, by reversing the sequence of differentiation in terms of space coordinates and time, we get,

$$\nabla.\vec{J}^i + \frac{\partial}{\partial t}\left(\nabla.\vec{D}\right) = 0$$

Using first Maxwell's equation for the second term, we have

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$$\nabla.\vec{J}^{i} + \frac{\partial \rho_v}{\partial t} = 0$$

2.5 Ampere's law modified by Maxwell for time-varying fields.

The static field Ampere's law is expressed as

$$\nabla \times \vec{H} = \vec{J}^{i}$$

Taking divergence of the above equation

$$\nabla.\nabla \times \vec{H} = \nabla.\vec{J}^{i}$$

From vector identity, the divergence of a curl is zero yielding

$$\nabla.\vec{J}^{i} = 0$$

Which contradicts the equation of continuity.

Maxwell deduced that Ampere's law for a static field must be changed to be applicable for a time-varying scenario since the equation of continuity is more fundamental than Ampere's circuital law by Belgacem [12]. Let the curl equation be modified by adding another vector $\vec{X}_j$ as $\nabla \times \vec{H} = \vec{J}^{i} + \vec{X}_j$

Taking divergence of both sides and using the vector identity divergence of curl is zero, one obtains $\nabla.\vec{J}^{i} + \nabla.\vec{X}_j = 0$

Using the equation of continuity, $\nabla.\vec{X}_j = \frac{\partial \rho_v}{\partial t}$

From Gauss law in differential form $\nabla.\vec{D} = \rho_v$, the above equation can be written as

$$\nabla.\vec{X}_j = \nabla.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

or, $\vec{X}_j = \frac{\partial \vec{D}}{\partial t}$

Ampere’s law can now be rewritten as

$$\nabla \times \vec{H} = \vec{J}^{i} + \frac{\partial \vec{D}}{\partial t}$$

The above equation is known Maxwell’s modification of Ampere’s law. The terms in the R.H.S. of the equation is the total current density consisting of circuit density $\vec{J}_C = \vec{J}^{i}$, and displacement current $J_D = \frac{\partial \vec{D}}{\partial t}$ generated due to time variation of the displacement vector $\vec{D}$.

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2.6 Maxwell's Equations in various forms:

In the absence of a magnetic field, a static $\vec{E}$ field can occur $\vec{H}$; an example is a capacitor with a static charge Q. Similarly, a constant current I conductor has a magnetic field $\vec{H}$ but no $\vec{E}$ field. However, when fields are time-varying, $\vec{H}$ cannot exist without an $\vec{E}$ field, nor can it exist without a corresponding $\vec{H}$ field. While static field theory can provide a wealth of useful information, it is only with time-varying fields that the full utility of electromagnetic field theory can be displayed. Faraday and Hertz's experiments, as well as Maxwell's theoretical analysis, all involved time-variable fields. The equations listed below, known as Maxwell's equations, were developed and analyzed individually in previous chapters. The most general form is shown in Table-1, where charges and conduction current may be present in the region. Under the divergence theorem, the differential and integral forms of the first two equations are equal.

Zhou [15] reviewed the independence, completeness of Maxwell's equation and uniqueness theorems in electromagnetics. He showed that the four Maxwell's equations are independent and complete. He showed that electrostatics and magnetostatics can be reduced from dynamical electromagnetics.

Table 1: Traditional form of Maxwell's Equations

Differential Form	**Integral Form**
$\nabla.\vec{D} = \rho_v$	$\oint_S \vec{D}.d\vec{S} = \int_v \rho_v dv$ (Gauss's law of electric)
$\nabla.\vec{B} = 0$	$\oint_S \vec{B}.d\vec{S} = 0$ (Gauss's law of magnetic)
$\nabla \times \vec{H} = \vec{J}_C + \frac{\partial \vec{D}}{\partial t}$	$\oint_S \vec{H}.d\vec{l} = \int_S \left(\vec{J}_C + \frac{\partial \vec{D}}{\partial t} \right) .d\vec{S}$ (Ampere's law)
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_S \vec{E}.d\vec{l} = \int_S \left(-\frac{\partial \vec{B}}{\partial t} \right) .d\vec{S}$ (Faraday's law, S fixed)

Now question is arise, are Maxwell's equation exist in free-space set? Do Maxwell's equations contain electric charge and magnetic field? We know that there is no electric charge and magnetic field in free space set. So, in free-space, where there are no charges $(\rho_v = 0)$ and no conduction currents $(J_C = 0)$ Maxwell's equations assume the form illustrated in Table-2.

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Table 2: Maxwell's Equations in Free Space

Differential Form	Integral Form
$\nabla.\vec{D} = 0$	$\oint_S \vec{D}.d\vec{S} = 0$
$\nabla.\vec{B} = 0$	$\oint_S \vec{B}.d\vec{S} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_S \vec{E}.d\vec{l} = \int_S \left(-\frac{\partial \vec{B}}{\partial t} \right).d\vec{S}$
$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$	$\oint_S \vec{H}.d\vec{l} = \int_S \left(\frac{\partial \vec{D}}{\partial t} \right).d\vec{S}$

The free-space set's first and second differential form equations can be used to demonstrate that time-variable $\vec{E}$ and $\vec{H}$ fields cannot exist independently. For example, if $\vec{E}$ is a function of time, $\vec{D} = \varepsilon_0 \vec{E}$ will also be a function of time, resulting in $\frac{\partial \vec{D}}{\partial t}$ being nonzero. As a result, $\nabla \times \vec{H}$ is nonzero, and a nonzero $\vec{H}$ must exist. Similarly, the second equation can be used to demonstrate that if $\vec{H}$ is a function of time, then a $\vec{E}$ field must exist. So, we can say that mathematical solution of Maxwell's equation are exist in free-space set.

2.7 Maxwell's Equations for Good Conductors

In a conducting medium with charge density ρ_v and conductivity σ, we can write, conducting current $\left(\vec{J}_C\right) >>$ displacement current $\left(\vec{J}_D = \frac{\partial \vec{D}}{\partial t} \right)$ (because σ is infinite and $\vec{J}_C = \sigma \vec{E}$).

Maxwell's equations are as given below:

$$\nabla.\vec{D} = \rho_v$$

$$\nabla.\vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}^{\,i}$$

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2.8 Maxwell's Equations for Non-conducting or Lossless Medium

In a lossless medium, current density $\vec{J}^i$ and charge density ρ_v are zero and Maxwell's equations are simplified as below:

$$\nabla.\vec{D} = 0$$

$$\nabla.\vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

2.9 Maxwell's Equations in Charge-Free/Current-Free Medium

There is no fee and no current if the material is lossless $(\sigma = 0)$.

This is referred to as a "perfect dielectric." An insulator is a dielectric that is good or near perfect, that is,

$$\nabla.\vec{E} = 0$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\nabla.\vec{H} = 0$$

$$\nabla \times \vec{H} = j\omega\varepsilon_0\vec{E}$$

2.10 Static Field Maxwell's Equations

The differential and integral forms of static field laws are summarized.

Differential Form	**Integral Form**
$\nabla.\vec{D} = \rho_v$	$\oint \vec{D}.d\vec{S} = \int \rho_v dv$ Charge is a $\vec{E}$ source sink.
$\nabla.\vec{B} = 0$	$\oint \vec{B}.d\vec{S} = 0$ Magnetic field lines are continuous, and there are no magnetic monopoles.
$\nabla \times \vec{E} = 0$	$\oint \vec{E}.d\vec{l} = 0$ Potential of a static $\vec{E}$ field depends on point rather than path rotation.
$\nabla \times \vec{H} = \vec{J}^i$	$\oint \vec{H}.d\vec{l} = \int \vec{J}^i.d\vec{S}$ Circuital law of Ampere.
$\nabla \times \vec{J}^i = 0$	$\oint \vec{J}^i.d\vec{S} = 0$ Constant current continuity equation.

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3. Discuss about Maxwell's Equations of free-space

3.1 Electromagnetic Wave Equation

Maxwell's equations are,

1. $\nabla.\vec{D} = 4\pi\rho$; where $\vec{D} =$ displacement vector, $\rho =$ charge density
2. $\nabla.\vec{B} = 0$; $\vec{B} =$ magnetic field induction
3. $\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$, $\vec{E} =$ electric intensity, $c =$ velocity of light
4. $\nabla \times \vec{H} = \frac{1}{c}4\pi\vec{J} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t}$; $J^i =$ current density, $\vec{H} =$ magnetic field intensity

But we know, $\vec{D} = K\vec{E}$, $\vec{J}^i = \delta\vec{E}$, and $\vec{B} = \mu\vec{H}$, $K =$ susceptibility, $\delta =$ conductivity, $\mu =$ permeability. For isotopic case, $\delta = \rho = 0$ and $\mu = K =$ constant.

Then the above equation becomes,

1. $\nabla.\vec{D} = \nabla.K\vec{E} = 0 \Rightarrow \nabla.\vec{E} = 0$
2. $\nabla.\vec{B} = \mu\vec{H} = 0 \Rightarrow \nabla.\vec{H} = 0$
3. $\nabla \times \vec{E} = -\frac{\mu}{c}\frac{\partial \vec{H}}{\partial t}$
4. $\nabla \times \vec{H} = 0 + \frac{K}{c}\frac{\partial \vec{E}}{\partial t}$

In free space $\mu = K = 1$ then the above equations becomes.

1. $\nabla.\vec{E} = 0$ (7)
2. $\nabla.\vec{H} = 0$ (8)
3. $\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}$ (9)
4. $\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$ (10)

Taking curl on both sides of equations (9), we get

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla \times \left(-\frac{1}{c}\frac{\partial \vec{H}}{\partial t}\right)$$

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$$\Rightarrow \nabla\times\left(\nabla\times\vec{E}\right)-\nabla^2\vec{E}=-\frac{1}{c}\frac{\partial}{\partial t}\left(\nabla\times\vec{H}\right)$$

$$\Rightarrow 0-\nabla^2\vec{E}=-\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow \nabla^2\vec{E}=\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} \qquad (11)$$

Similarly taking curl on the both sides of equation (10) and then we get,

$$\nabla^2\vec{H}=\frac{1}{c^2}\frac{\partial\vec{H}}{\partial t} \qquad (12)$$

Then clearly (11) and (12) satisfy wave equation,

$$\nabla^2\psi=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

The wave velocity $v=c=$ velocity of light $=3\times10^8$ m/s. Hence we conclude that the light is an electromagnetic phenomena.

3.2 The Poynting Vector or Radiant Theorem

We know from Maxwell's equation,

$$\nabla\times\vec{H}=\vec{J}^{\,i}+\frac{\partial\vec{D}}{\partial t}$$

$$\Rightarrow \nabla\times\vec{H}=\vec{J}^{\,i}+\frac{\partial\vec{D}}{\partial t}$$

$$\Rightarrow \vec{J}^{\,i}=\nabla\times\vec{H}-\frac{\partial\vec{D}}{\partial t}$$

$$\Rightarrow \vec{J}^{\,i}=\nabla\times\vec{H}-\frac{\partial}{\partial t}\left(K\vec{E}\right)\therefore\left[\vec{D}=K\vec{E}\right]$$

$$\therefore \vec{J}^{\,i}=\nabla\times\vec{H}-K\frac{\partial\vec{E}}{\partial t}$$

Taking $\vec{E}$ on both sides, we get

$$E.\vec{J}^{\,i}=\vec{E}.\left(\nabla\times\vec{H}\right)-K\vec{E}\frac{\partial\vec{E}}{\partial t} \qquad (13)$$

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Now,

$$\nabla.\left(\vec{E}\times\vec{H}\right)=\vec{H}.curl\vec{E}-\vec{E}.curl\vec{H}$$

Using this in (1), we get

$$E.\vec{J}=\vec{H}.curl\vec{E}-\nabla.\left(\vec{E}\times\vec{H}\right)-K\vec{E}\frac{\partial\vec{E}}{\partial t} \qquad (14)$$

Taking volume integral on both sides of equations (14), we get

$$\int_v \vec{E}.\vec{J}dv=-\frac{1}{2}\int_v\frac{\partial}{\partial t}\left(\mu\vec{H}^2+K\vec{E}^2\right)dv-\int_v div\left(\vec{E}\times\vec{H}\right)dv$$

$$=-\frac{\partial}{\partial t}\int_v\left(\frac{\mu\vec{H}^2}{2}+\frac{K\vec{E}^2}{2}\right)dv-\oint_s\left(\vec{E}\times\vec{H}\right)ds \qquad (15)$$

Each of the term in the above equation is capable of physical interpretation.

$\int_v \vec{E}.\vec{J}dv$ gives the rate dissipation of energy within volume V. The first term of R.H.S. of equation (15) can be interpreted when it is realizes that $\frac{\mu\vec{H}^2}{2}$ is the energy stores per unit volume in the magnetic field and $\frac{K\vec{E}^2}{2}$ is the energy stores per unit volume in electric intensity. The final item, the closed surface integral of $\vec{E}\times\vec{H}$, can be thought of as the power that flows out via a surface S. The vector as a vector quantity such that it is integral over a closed surface S. hence it is power flow per unit area $\left(\vec{E}\times\vec{H}\right)$. This vector is usually called Poynting vector. i.e, $\vec{P}=\vec{E}\times\vec{H}$.

4. Conclusion:

This paper provided an illustration of four recognized law and goes through how to derive them as an expansion of Maxwell's equations. Furthermore we have interpreted the solution of Maxwell's equations solution, by incorporating the general set and free-space set mathematically. The Maxwell's equations place an important providing mathematical model for electric optical and radio technologies such as power generation, electric motor, wireless communication, lenses, radar etc. We also verified that the light is an electromagnetic phenomenon. The combination of Maxwell's equations, boundary equations and constitutive relations show everything that can be experimentally measured about electromagnetic effects.

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